The Braess Paradox and Coordination Failure in Directed Networks with Mixed Externalities

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Abstract

The Braess Paradox (BP) illustrates an important counterintuitive observation that adding links to a directed transportation network with usage externalities may raise the costs of all users. Research on the BP traditionally focuses on congestible networks. We propose and experimentally test a new and more dramatic version of the BP, where the network exhibits both congestion (negative externalities) and cost-sharing (positive externalities) characteristics. Our design also involves experimental manipulation of choice observability, where players choose routes simultaneously in one condition and sequentially in the other. We report robust behavioral evidence of the BP in both conditions. In nine of 10 sessions in the basic network, subjects coordinated successfully to achieve the welfare-maximizing equilibrium. But once the network was augmented with a new link, coordination failure resulted in a major proportion of subjects switching to a new route, resulting in a 37% average *increase* in individual travel cost across conditions.

Keywords: Braess Paradox; Transportation networks; Positive and negative network externalities; Choice observability; Coordination; Experiments; Behavioral operations

1. Introduction

Transportation networks constitute common examples of directed networks. Their study is an important topic in supply chain management as different supply chains make use of the same network infrastructure together with other passengers and drivers, typically in a decentralized manner (see e.g., Cominetti, Correa, and Stier-Moses 2006, 2009). A major characteristic of these networks is the presence of externalities, such as congestion externalities, among users traversing shared links. It might seem quite natural to assume that adding one or more links to such a network would never increase social costs, and would generally decrease social costs at the benefit of the users. Braess (1968) has shattered this belief by illustrating that, paradoxically, augmenting a congestible network by adding one or more links may increase the equilibrium costs of *all* the users. An alternative implication of this finding is that eliminating one or more links may, in fact, benefit all the users. This highly counterintuitive phenomenon, called the *Braess Paradox* (BP), has instigated considerable investigations, both theoretical and experimental, mostly in computer science but also in transportation research and behavioral economics (see Roughgarden 2005, Nisan et al. 2007).

In this paper, we introduce a new setting for the BP that involves positive (cost-sharing) as well as negative (congestion) network externalities at different levels of choice observability among network users. We study our new setting in a laboratory experiment and find that, over and above theoretical predictions of BP-type welfare loss, coordination failure among experimental participants lead to further drastic deterioration in welfare upon network augmentation. These developments contribute significantly to a line of behavioral studies on directed networks that are conducive to the BP, such as Rapoport et al. (2009), Morgan, Orzen, and Sefton (2009), and Rapoport, Gisches, and Mak (2014).

A necessary condition for the occurrence of the BP is that equilibrium traffic in the augmented network is inefficient, so that the augmented network exhibits a social dilemma (Mak and Rapoport 2013). In the social sciences, particularly in economics and psychology, the best known and most extensively studied examples of social dilemmas in non-cooperative games include the Prisoners' Dilemma (e.g., Luce and Raiffa 1957), public goods games (e.g., Ledyard 1995), and variants of Centipede game (e.g., Aumann 1995, 1996, and Binmore 1996, for theoretical discussions, and McKelvey and Palfrey 1992 for experimental investigation). The BP is distinct from these standard forms of social dilemma in terms of the following two features:

(1) The BP applies to route choices in directed networks, which encompass a wide range of strategic interactions (Rosenthal 1981) that are highly distinctive in themselves. In practice, the BP has potentially important implications for the design of transportation and telecommunications networks that are not matched by other common social dilemma games,

(2) The BP demonstrates how a non-trivial yet simple *structural* change in a game of strategic interactions, in the form of an expansion of the players' strategy sets, may lead to the welfare-maximizing state being not an equilibrium anymore, *and* at the same time the appearance of new, Pareto inefficient equilibria with inferior social costs. For example, the BP introduced in this paper includes adding only a single link to a basic network, which results in an expansion of the strategy set of every network user by one new route. This expansion leads to the welfare-maximizing traffic, an equilibrium outcome in the basic network, becoming not an equilibrium anymore. At the same time, new equilibria appear that involve a major proportion of network users migrating to the new route, causing significant negative impact on efficiency. Importantly, the original and the augmented networks have multiple equilibria, and a drastic deterioration in welfare in our experiment occurred partly because of coordination failure in the augmented network. To our knowledge, no analogous effects have been studied with standard forms of social dilemmas.

Although the BP has been discussed largely in computer science and transportation science, it is generally important in investigating social costs in networks with usage externalities (e.g., Vickrey 1969; Daniel, Gisches, and Rapoport 2009). Moreover, whereas experimental research on other forms of social dilemmas would seem to have reached its peak, experimental research on the BP is still in its infancy. There is still much to understand about the BP, especially as a behavioral phenomenon.

1.1. Positive and Negative Externalities: Cost-sharing vs. Congestion

Studies on the Braess Paradox have focused mostly on congestible networks with negative network externalities. Positive network externalities exist if the user's benefits are an increasing function of the number of other network users, whereas negative network externalities exist if they constitute a decreasing function. In studies of choice of routes in directed networks, negative externalities are typically generated by the additional delay in travel time imposed on other network users, exhaust emissions, and noise. Positive externalities can be generated by modes of public transportation (e.g., carpool, shuttle) where the cost of travel is divided – equally or unequally – among the users. As noted by, e.g., Easley and Kleinberg (2010) and Holzman and Monderer (2015), the analysis of route choice games with either positive or negative externalities might be quite different. After comparing these two classes of games, Easley and Kleinberg conclude that "with positive externalities, there exist self-fulfilling expectations and a natural set of outcomes to coordinate on; with negative externalities, any shared expectation of a fixed audience size will be self-negating, and the individuals must instead sort themselves out in much more complicated ways" (2010, p. 536). To further complicate matters, it is not necessarily the case that all the link cost functions are either monotonically increasing or decreasing. Easley and Kleinberg provide an example of an online social media site, where a limited infrastructure might be most enjoyable if it has a reasonably large audience (positive externalities), but not very enjoyable if the audience is so large that connecting it to the website becomes very slow because of congestion (negative externalities). In their example, it is the *total number* of network users that may generate two different types of externalities with no change in the topology of the network.

In contrast, we examine below another class of scenarios where the total number of network users is fixed and commonly known, but the *architecture* of the network is changed. Under the new architecture, some links generate negative externalities whereas other links generate positive externalities, resulting in a network with mixed (network) externalities. Our scenario is motivated by decentralized traffic networks with a fixed population size, where individual users may independently choose between traversing the network in their own private cars, thereby generating congestion on the road (and consequently longer travel delays), or by means of cost-sharing public transportation (e.g., carpool), where the cost of travel is shared by all the users choosing the same mode of transportation.

Fundamentally, our incorporation of positive externalities lead to multiple equilibria and coordination problems, as Easley and Kleinberg (2010) have also noted. The new type of the BP that we study differs markedly from previous BP settings – which only involve negative externalities – because of this new characteristic. In previous BP settings such as Rapoport et al. (2009), the BP occurred because the (typically unique) equilibrium changed. Here it could happen because of coordination failure as a behavioral phenomenon. This is especially the case in our setup when players make decision simultaneously, so that theoretically the multiplicity of equilibria allows for different possibility of cost changes. The equilibrium changes caused by the augmentation of the network would not have resulted in as drastic a deterioration in welfare as observed in our experiment, if users coordinated their strategy choices to achieve the welfare-maximizing (Pareto dominant) equilibrium in both the basic and augmented networks. As such, our study also contributes to the voluminous theoretical and experimental literature on coordination games and equilibrium selection (e.g., Schelling 1960, 2006, Harsanyi and Selten 1988, Cooper 1999, Van Huyck, Battalio, and Beil 1990, 1991).

1.2. Observability of Other Users' Route Choices: Simultaneous vs. Sequential Protocols

In all previous experiments on the Braess Paradox, every user committed him/herself independently to a route at the outset with no opportunity to change it *en route*. We refer to this procedure as the *simultaneous protocol* of play. In the present study, we investigate experimentally both the simultaneous protocol and the *sequential protocol* of play. Under the latter protocol, the network users choose routes sequentially in an exogenously determined order of play, with complete information about the route choices of all the users preceding them in the sequence. Intuitively, the sequential protocol might be expected to enhance coordination among users and improve efficiency. However, previous research on choice observability under different domains of sequential actions, such as effort contributions in teams (Winter 2006, 2010, Klor et al. 2014) and public good contributions (Varian 1994, Vesterlund 2003, Steiger and Zultan 2014), offers mixed evidence. In fact, they demonstrate that higher choice observability might lead to lower efficiency in some cases. We shall investigate analogous issues in our network context.

The present study has two main purposes. Our first purpose is to construct a directed network that includes a link with positive externalities and two other links with negative externalities; we then test the hypothesis that the BP is realized when the original basic network is expanded by connecting the two links that generate negative externalities with a new link to form an augmented network. As it turns out, theoretically, the BP in our setup is even more dramatic (in terms of the relative loss of welfare as the network is augmented) than comparable versions with only congestion externalities and affine cost functions. Our second purpose is to compare route choice in the basic and augmented networks under the simultaneous and sequential protocols of play, which represent two extremes of choice observability.

For both purposes, we conducted laboratory experiments to examine the behavioral validity of the BP in our network settings. We note (see, e.g., Roughgarden 2005) that traffic patterns may change quickly and sometimes significantly even when the same population of users traverse the network many times. Therefore, the assumption of a static model is questionable. We are interested in determining whether players converge to equilibrium with experience, and if so, how quickly they do so, or whether they deviate from an inefficient equilibrium after a few iterations in order to enhance their social welfare. Thus, following common practices in experimental economics, we iterated both the basic and augmented network games multiple times in our experiment.

The theoretical and experimental evidence reported in this study points to several substantial contributions to the study of the BP. Firstly, we offer a radically different theoretical structure for our version of BP compared with previous structures. The equilibria are different, and the scenario takes on the characteristics of different coordination games in different networks, leading to different equilibrium selection issues. It is a possibility in our theoretical analysis, as well as an empirically observable fact in our experiment (the conditions with the simultaneous protocol), that welfare loss can be aggravated by coordination failure as a behavioral phenomenon if the network is augmented. In this sense, we have made substantial theoretical and behavioral contribution to network games. The basic BP of Rapoport et al. (2009) showed experimentally that when a network was augmented, the equilibrium changed and the BP arose when network users followed equilibrium play. The present study shows that augmenting a network with multiple equilibria leads to coordination failure and further deterioration in social welfare. Note that, in our framework, the multiplicity is eliminated under the sequential protocol, or when the link with positive externalities is removed leaving only a standard BP setting. But, as shall be seen, in our experiment the sequential protocol brought about convergence to equilibrium to significantly different behavioral extents under the basic network compared with the augmented one. Thus, we establish additional theoretical and behavioral contributions in our study of the sequential protocol.

The rest of the paper is organized as follows. Section 2 reviews experimental literature on the BP. Section 3 presents the basic and augmented networks and their equilibrium analysis. The experimental method and results are described in detail in Sections 4 and 5, respectively. Section 6 concludes with a brief discussion of our findings.

2. Empirical Evidence and Experimental Research on the Braess Paradox

There is anecdotal evidence that the Braess Paradox might have occurred in several major cities in the USA, Europe, and Asia. Murchland (1970) remarked briefly that, in agreement with the implications of the BP, major road improvements in the center of Stuttgart in Germany had not yielded the benefits expected. A later article published in *New York Times* with the provocative title "What if they closed 42nd street and nobody noticed" (Kolata 1995), hinted at the counterintuitive consequences of road closure. More systematic evidence was subsequently reported by Youn et al. (2008), who demonstrated by computer simulations specific routes in Boston, London, and New York City where the BP might actually occur, and pointed out roads that might have been closed to reduce the predicted adverse effects of the BP. More recently, an international team of researchers from France and Belgium published a *Physical Review Letters* article claiming that the BP might occur in mesoscopic electron systems (Pala et al. 2012); in particular, they reported that adding a path for electrons in mesoscopic network paradoxically reduced its conductance. Moreover, our study pertains to the possibility of coordination failure in networks; this is a very realistic problem in traffic networks, which by its complexity should be expected to often allow for multiple equilibria.

While the above empirical and simulation evidence supports the claim that the BP is more than a theoretical curiosity, it cannot answer questions about the impact of experience and learning on human users' choice of routes over time. To do so, the assumptions underlying the BP about the cost structure and population size have to be properly operationalized for behaviorial investigation. A complementary approach is required that simulates in the laboratory directed networks, and then observes whether choice behavior supports equilibrium play. As pointed out by Gisches and Rapoport (2012), a major advantage of this approach is experimental control, while major disadvantages are relatively small groups of users and small financial stakes. The results of controlled experiments complement the anecdotal and empirical evidence rather than replace them.

Several studies of the BP have manipulated experimentally the following independent variables: group size, network architecture, symmetry vs. asymmetry of the network users, and type of information about route choices of the group members at the end of each round of play. The study by Rapoport et al. (2009), which sets the foundation for this paper, presented in their Games 1A and 1B (which were used in their Experiment 1) a group of n = 18 players in two directed networks that are displayed in Figure 1. In one of their conditions, the players first chose one of the two routes in 40 iterations of the basic network in Figure 1(a). Afterwards, they were handed a new set of instructions and simultaneously chose one of the three routes in the augmented network in Figure 1(b). In a second condition with different players, the order of presentations of the two networks in Figure 1(a) and 1(b) was reversed. In both conditions, route choices in the augmented network (Figure 1(b)) converged slowly to (a unique) equilibrium over 40 rounds. A major finding is that the order of the networks had no discernible effect on the route choices.

[Insert Figures 1 and 2 about here]

Additional experimental studies of the BP have been reported by Rapoport, Mak, and Zwick (2006), Rapoport et al. (2008), Morgan et al. (2009), Gisches and Rapoport (2012), Dal Forno and Merlone (2013b), and Rapoport et al. (2014). Rapoport et al. (2006) demonstrated how a change in the user population size could affect the occurrence of the BP. Rapoport et al. (2008) constructed two networks with a richer architecture than the network in Figure 1(a), but which are also susceptible to the BP. They report significant evidence of the BP although not as strong as in the previous simpler networks. Studying a considerably richer network than in Figure 1(a) with six routes in the augmented network, Gisches and Rapoport (2012) investigated the BP under two information structures: public monitoring, where each player is accurately informed of the route choices and payoffs of all the players in the previous round, and private monitoring, where each player is only informed of her own payoff in the previous round. Learning was considerably facilitated under public rather than private monitoring. Evidence in support of the BP was reported in both information conditions.

3. The Network Games

3.1. Notation, Terminology, and Design

We consider directed networks with common origin and common destination that are modeled by a graph $G(V, E)$, with a vertex (node) set V, link (edge, arc) set E, and a set $K \subseteq V \times V$ of origin-destination routes. The network serves a finite and commonly known number of players (users) n. Under the simultaneous protocol of play, each player i independently chooses a single route (path) p_i from the origin O to the destination D in order to minimize her travel cost C_i. Under the sequential protocol of play, players choose routes in an exogenously determined order. When it is her turn to choose a route, player i is fully informed of the route choices of all the players who preceded her in the sequence. The cost of route p_i to player i depends on all the n routes, and may either be increasing or decreasing with the number of players choosing this route.

Denote by f_{jk} the number of players who choose the link jk from vertex j to vertex k in the network. The cost incurred by player i in choosing link jk is denoted by $c_i(f_{jk})$. As is common in the literature on transportation, the cost $c_i(f_{jk})$ is used as a proxy for the delay in traversing the link jk. We assume that $c_i(f_{jk})$ is the same for all players. The total cost for choosing a route is the sum of the link costs over all the links in this route.

We consider two classes of links depending on whether $c_i(f_{jk})$ is either increasing or decreasing in f_{jk}. Increasing costs are modeled in our study with an affine function, where for each link jk, $c_i(f_{jk}) = a_{jk}f_{jk} + b_{jk}$, and a_{jk}, $b_{jk} \geq 0$. The fixed constant b_{jk} is interpreted (Rapoport et al. 2009) as the minimum delay to traverse link jk in the absence of traffic, whereas the variable component a_{jk} models the effect of congestion. Affine cost functions are chosen because they provide good approximation of reality (Steinberg and Zangwill 1983); importantly, they are also easy to explain to inexperienced subjects in network game experiments.

Decreasing costs are modeled in our study with the function $c_i(f_{jk}) = a_{jk}/f_{jk} + b_{jk}$, and a_{jk}, $b_{jk} \geq 0$. That is, part of the cost of the link is shared equally among players traversing that link (cf. Liu, Mak, and Rapoport 2015). This equal ("fair") cost-sharing allocation mechanism can be rationalized by economic theorizing in the following ways: (1) it can be derived as a group bargaining outcome (not explicitly modelled in the current framework) using the Shapley value (cf. Moulin and Shenker 2001, and Chen and Roughgarden 2009); and (2) it can be shown to be the unique cost-sharing scheme that satisfies a set of different axioms (see Feigenbaum, Papadimitriou, and Shenker 2001, and Herzog, Shenker, and Estrin 1997). Figure 2(a) exhibits the network presented to our players. The cost structure is given by:

$a_{OA} = a_{BD} = 1$, $b_{OA} = b_{BD} = 0$,

$a_{AD} = a_{OB} = 0$, $b_{AD} = b_{OB} = 21$,

$a_{OD} = 90$, $b_{OD} = 10$.

Henceforth, we shall simplify notation so that OAD stands for the two-link route from O to D via A, OD stands for the single-link route from O to D, etc. Clearly, OAD and OBD are congestible, whereas the single-link OD calls for equal cost sharing.

To fix ideas, we build on Games 1A and 1B in Rapoport et al. (2009) (Figure 1) to construct the networks in Figure 2 as stylized systems that allow for both private and public transportation. Routes OAD and OBD model a combination of private car driving on congestible roads and public transportation on roads not susceptible to congestion. Route OD models an alternative public transportation from the origin to the destination with equal sharing of the cost of travel. It is straightforward to verify that all players traversing OD is the welfare-maximizing traffic that minimizes individual travel costs.

The augmented network (Figure 2 (b)) is constructed by connecting the two congestible links (OA) and (BD) with a new link (AB), where $c_i(f_{AB}) \equiv 1$. In the augmented network, the player is faced with a choice among *four* origin-destination routes, namely, the original three routes OAD, OBD, and OD, and a new route OABD. It is no longer apparent what distribution of route choices to expect under either of the two protocols.

As might also be expected, the augmented network presents a potential for welfare improvement. The welfare-maximizing traffic in that network has 16 players traversing OD and two players traversing OABD (see Appendix A), resulting in an average individual travel cost of 14.4, which is superior to the welfare-maximizing individual travel cost of 15 in the basic network with all the *n* players choosing the direct route OD. But as discussed below, the welfare-maximizing traffic in the augmented network is no longer an equilibrium, nor even the traffic with all the *n* players traversing OD. Instead, equilibrium traffic in the augmented network results in worse welfare outcomes their either traffic pattern.

3.2. Equilibrium Analysis

[Insert Table 1 about here]

We proceed to construct equilibrium solutions for both networks (basic vs. augmented) under each of the two protocols (simultaneous vs. sequential), focusing on pure-strategy and symmetric mixed-strategy equilibria. Note that our analysis is based on the Nash equilibrium concept, rather than the concept of Wardrop equilibrium, which is common in transportation science (see Wardrop, 1952, Beckmann et al., 1956, Correa & Stier-Moses, 2010). In a Wardrop equilibrium, all users incur the same travel cost, and the travel cost of any used route is less than or equal to any unused route. It is a Nash equilibrium when its number of users are all integers. In general, the Nash equilibria converge to the Wardrop equilibria when the group size is large (see e.g., Morgan et al., 2009). Table 1 summarizes our results by listing the traffic in the feasible equilibria. Details of the derivations are presented in Appendix A.

For comparisons, we have also listed in Table 1 the equilibria in the networks in Figure 1, i.e., Games 1A and 1B in Rapoport et al. (2009). Rapoport et al. (2009) only involves the simultaneous protocol, but as discussed in Appendix A, the equilibria for those games in the sequential protocol have the same traffic and costs as the corresponding pure-strategy equilibria under the simultaneous protocol. It appears that, compared with Rapoport et al.'s networks, the addition of the positive externalities route OD introduce *additional* equilibria under the simultaneous protocol, and thus equilibrium selection and coordination issues. Under the sequential protocol, the number of equilibria has not increased, but the equilibria are completely different.

3.2.1. Simultaneous Protocol. For the basic network, as listed in Table 1(a), there are two pure-strategy equilibrium traffic flows: (1) the 18 players being splitted equally between routes OAD and OBD, and (2) the welfare-maximizing traffic of all players traversing route OD. There are two symmetric mixed-strategy equilibria, which are listed in Table 1(b): (1) each player chooses OD with probability 0.27 and the other two routes with equal probabilities (0.36), and (2) each player chooses OAD and OBD with equal probabilities of 0.5.

In the augmented network, routes OAD and OBD are strictly dominated by OABD and therefore are not chosen in equilibrium. There are three pure-strategy equilibrium traffic flows (Table 1(a)), in which the numbers of players traversing OD are respectively 10, 9, and 0. Note that the welfare-maximizing traffic of 16 choosing OD and two choosing OABD is not an equilibrium in the augmented network. The intuition is that OABD has very low cost when there is low traffic on it, due to the fact that its cost is almost entirely driven by the negative externalities of congestion, unlike OAD and OBD. Therefore, any traffic with very few or zero players on OABD (as is the case with the welfare-maximizing traffic) would attract switching from players on other routes, and cannot be an equilibrium. Symmetric mixed-strategy equilibria in the augmented network must assign zero probability to OAD and OBD, since they are strictly dominated by OABD. Our computation finds one mixed-strategy equilibrium (Table 1(b)): each player chooses OD with probability 0.29 and OABD with probability 0.71.

3.2.2. Sequential Protocol. The general approach that we use in analyzing the sequential protocol is backward induction. As listed in Table 1(c), under this protocol, the unique subgame perfect equilibrium for the basic network is for all players to choose OD – the welfare-maximizing traffic. Meanwhile, the unique (up to a trembling hand refinement) subgame perfect equilibrium for the augmented network is for the first nine players to choose OABD and the last nine players to choose OD; intuitively, the early players in the sequence choose OABD when it is not highly congested, but after congestion on that route builds up due to the early players' choices, the later players choose the cost-sharing route OD. As with the simultaneous protocol, the welfare-maximizing traffic is not an equilibrium in the augmented network under the sequential protocol.

3.2.3. The Braess Paradox in the Present Networks. Regardless of protocol, the welfare-maximizing traffic is an equilibrium in the basic but *not* in the augmented network. This implies that traffic in these networks could potentially exhibit the BP, namely, that augmenting the basic network leads to a loss of welfare among the players in equilibrium. In the worst case, under the simultaneous protocol, the individual equilibrium travel cost increases from 15 to 37, a 46.7% increase, when the network becomes augmented. The corresponding (unique) increase in average equilibrium travel cost under the sequential protocol is from 15 to 19.5, a 30% increase.

The BP arises in our networks because players who all traverse OD in the basic network equilibrium would partially switch to OABD, a newly available route in the augmented network equilibrium; the switch is individually optimal but collectively disadvantageous. It is instructive to compare this new version of the BP with a standard version with only congestible cost functions (negative network externalities). Here, a convenient benchmark is the same two networks *without* OD, where it is straightforward to show that augmenting the network leads to a change in equilibrium individual travel cost from 30 to 37, or a 23.3% increase. This is a notably less severe loss in welfare than when the positive network externalities of the OD route are present. Even though positive externalities exist in identical form in both networks in our setup, those externalities are exploited much more fully in the basic than in the augmented network, leading to a more dramatic form of the BP than in a standard version.

Under the sequential protocol, the equilibria in both networks are unique. The sequential protocol eliminates the multiplicity and seems to drastically improve convergence in the basic network, but not much in the augmented network. The intuition behind this is that the sequential nature of the game is conducive to "bandwagon" effects that are helpful in coordinating players to build up choices of the cost-sharing route, which particularly narrows down the feasible equilibria in the basic network (though *not* augmented network) to the welfare-maximizing traffic. However, under the simultaneous protocol, whether a BP-type welfare loss takes place depends on which of the multiple pure- and mixed-strategy equilibria in each network is played out. As we shall see, this outcome can be sensitive to individual player characteristics (cf. Dal Forno and Merlone 2013a). If, in the basic network, a pure-strategy equilibrium with equal split between OAD and OBD is played out, while in the augmented network a pure-strategy equilibrium with 9 or 10 players traversing OD is played out (see Table 1(a)), then the augmented network will exhibit an improvement in welfare over the basic network. Likewise, if symmetric mixed-strategy equilibria are played out in both networks, then there also would be an improvement in welfare by augmenting the basic network. Otherwise, the BP will be observed.

Our discussion above implies a behavioral question: would the BP be robustly observed for both protocols with human players? We seek an answer to this question through our experiment, as reported in the following sections.

4. The Experiment

4.1. Subjects

One hundred and eighty undergraduate students volunteered to participate in a computer-controlled experiment on decision making for payoff contingent on performance. Male and female students participated in almost equal numbers. The subjects were assigned into ten groups. Five groups of 18 subjects each were assigned to Condition SIM, where they were instructed to play the basic and augmented games under the simultaneous protocol of play. Five additional groups of 18 subjects each were assigned to Condition SEQ, where they were instructed to play the basic and augmented games under the sequential protocol of play. Payments were cumulative and stated in tokens that were converted to US dollars at the end of the session. Excluding a $5 show-up fee, the mean payoff across all ten sessions was $24.5. On average, each session lasted approximately two hours.

4.2. Procedure

The experimental sessions were conducted in research laboratories with multiple terminals located in separate cubicles. Subjects were first handed the instructions for Part I (see Appendix B for the instructions in Condition SEQ) that they read at their own pace. Questions about the procedure were answered individually by the experimenter.

Each session was divided into Parts I and II. The instructions for Part I presented and explained the basic network game in Figure 2(a), and the instructions for Part II did the same for the augmented network game in Figure 2(b). In Part I, the subjects in all the 10 sessions first chose origin-destination routes in the basic network game that was repeated multiple times. Only after completing Part I, they were handed a new set of instructions for Part II and asked to choose routes in the augmented network. Because play in Condition SEQ took longer time, the number of rounds in each part was different, so that subjects played 40 rounds of each network game (basic followed by augmented) in Condition SIM but only 20 rounds of each game in Condition SEQ. Each set of instructions exhibited the network, explained the cost functions, illustrated the computation of the travel cost, and described the procedure for choosing a route. After all the n members of each group chose their routes, a new screen was displayed with full information presented about the number of players choosing each route and their payoff for the round.

Rather than charging the subject his/her cost of travel, the payoff for each round was computed by *subtracting* the cost from a fixed reward R awarded to the subject for completing the trip. The value of R in both conditions was set at 45 tokens, but the conversion of tokens to dollars was 75:1 in Condition SIM and 45:1 in Condition SEQ to account for the difference in number of rounds between the two conditions. The value of R was judiciously chosen so as to potentially create a significant difference in payoffs with the two networks. For example, in the worst equilibria in Condition SIM, a subject would earn 45 - 30 = 15 tokens in Part I and only 45 - 37 = 8 tokens in Part II. The former payoff being almost double the latter payoff, this is a more dramatic change than the costs (30 vs. 37) because of the across-the-board reward of $R = 45$ tokens.

Two features of the design warrant discussion. First, we have opted for a within-subjects design in each of the two conditions with subjects playing first the basic and then the augmented network games in this order. This feature renders it considerably more difficult to generate evidence in support of the BP because subjects would first be exposed to the welfare-maximizing traffic as a rational equilibrium possibility before the network is augmented. If the effect of the BP is realized under a within-subjects design with the same subjects participating in both network games beginning with the basic network, then the effect should prove much more convincing. Second, as discussed in the previous paragraph, the adverse outcome of the BP has been considerably *amplified* by assigning the reward of 45 tokens to each subject on each round in both parts of the session, thereby framing the network games in terms of (typically) gains rather than losses.

5. Results

5.1. Preliminary Observations

Figure 3 displays the route choice distributions for all 10 sessions in Conditions SIM and SEQ. Note that the two graphs on the same row are from the same session – subjects played first in the basic network (left panels) followed by the augmented network (right panels).

In the basic network rounds, in nine of the 10 sessions across both conditions subjects converged towards the welfare-maximizing equilibrium traffic of all traversing the cost-sharing route OD. However, in one session in Condition SIM (bottom row left column entry in Figure 3(a)), traffic approximated the equilibrium with equal split between OAD and OBD. While this is a single exception in our experiment, it suggests that traffic in Condition SIM was susceptible to bifurcation between multiple equilibria (Dal Forno and Merlone 2013a). We shall offer further discussion on this point in Section 5.4. Otherwise, in most of the following analysis, we include the exceptional session in our statistical tests that aim to demonstrate the Braess paradox. Including that session would only tend to decrease the power of such tests, as welfare generally increased in that particular session when the network was augmented. Excluding the session does not change our major conclusions. In this sense, we have adopted a conservative approach in reporting the data analysis for our main purpose.

[Insert Figure 3 and Table 2 about here]

Once the network was augmented, a substantial proportion of subjects in every session switched to OABD, leading to much instability that persisted throughout the augmented network rounds. It is also worth noting that, in keeping with OAD and OBD being strictly dominated by OABD in the augmented network, the choice frequencies of OAD and OBD indeed became negligible when the subjects chose their routes in that network.

The aggregate statistics in Table 2 confirm the observations from Figure 3. Subjects largely chose OD in the basic network rounds, regardless of the choice observability. However, they split their route choices between OABD and OD in the augmented network rounds, slightly favoring OABD. We also tested statistically whether the observed route choice frequencies and travel costs converge towards any equilibrium predictions in Table 1. Details are presented in Appendix C. Although we have tested against all predictions in Table 1, preliminary observations and subsequent analysis show that the most relevant predictions for comparisons are those listed in Table 2. For comparisons, we have also listed (in rows 11-13) the results from Rapoport et al. (2009)'s Experiment 1. It can be seen that the presence of the positive externalities route OD in our experiment led to very different traffic patterns from Rapoport et al.'s experiment.

Overall, our conclusions are consistent with the observations exhibited in Figure 3: in every session except one, route choice distributions and travel costs in the basic network rounds converged towards the welfare-maximizing equilibrium of all choosing OD. However, in the exceptional session in Condition SIM, route choice distributions were statistically not different from the equilibrium traffic with equal split of all subjects across OAD and OBD.

By contrast, as detailed in Appendix C, traffic and travel costs in the augmented network rounds exhibited significant statistical deviations from all equilibrium *point* predictions in Condition SIM, although their means lie between the point predictions. But in Condition SEQ, these variables statistically converged towards the equilibrium traffic of equal split across OD and OABD. This is largely consistent with our intuition that subjects in Condition SEQ would be relatively able to coordinate among themselves to achieve equilibrium traffic.

5.2. Behavioral Evidence of the Braess Paradox

The cost statistics in Table 2 clearly show that travel costs *increased* in general when the network was augmented: the mean travel cost aggregated over all the rounds and subjects within the same condition increased from 19.2 to 24.1 as the network was augmented in Condition SIM; the corresponding increase in Condition SEQ was from 15.5 to 22.6.

We also work out each subject's percentage change in travel cost as the network was augmented, and then average those percentage changes for each condition. As it turns out, the increase in individual travel costs as the network was augmented was, on average, 32% and 46% in Conditions SIM and SEQ, respectively (the percentage increase in Condition SIM is the same if we limit analysis to the first 20 rounds only for direct comparison with Condition SEQ). The overall mean proportional increase was 37% across conditions.

The inefficiency of these changes may also be appreciated by means of the following correlational analysis: we find that the proportional frequency of subjects choosing OD in a session correlated negatively with the average travel cost among all subjects in the same session at high correlation coefficients that exceed 0.99 ($p<0.01$) in both conditions and in both networks. That is, overall, choosing OD did lead to significantly decreased social costs. Yet many subjects in the augmented network rounds migrated to OABD for individual gains.

To further illustrate our general observations, we report a mixed-design 2(choice observability: Condition SIM vs. Condition SEQ) × 2(network: basic vs. augmented) ANOVA, with choice observability as the between-subjects and network as the within-subjects factors, while the overall travel cost is the dependent variable. The unit of observation is the session. The analysis reveals a significant main effect in network ($F(1,8) = 25.1$, $p < 0.01$) but no other significant effects at $p < 0.05$. This is consistent with our other observations and indicates that, across both conditions, our experiment exhibited the BP to similar extent.

Note that Condition SIM had a seemingly lower welfare loss, partly because the sessions had more rounds allowing for more learning. Moreover, as noted earlier, one session in that condition did not converge to the welfare-maximizing traffic in the basic network rounds; that session did not exhibit the BP, with average per round travel cost *decreasing* slightly from 31.1 to 29.9 as the network was augmented, in contrast with the other sessions (cf. Section 5.4).

In the following sections, we discuss several sets of additional analysis that reveal further insights into our experimental data – most notably coordination failure in the augmented network rounds.

5.3. Coordination Failure and Fluctuations of Choices in the Augmented Network Rounds

Coordination failure in an experimental game typically means players converging to an inefficient (Pareto dominated) equilibrium; in looser terms, coordination failure could also refer to players failing to converge to any equilibrium at all. By both notions, our experimental data from the augmented network rounds under both Conditions SIM and SEQ display coordination failure, especially when compared with behavior in the basic network rounds. In the basic network rounds, all but one of the sessions in Condition SIM and all sessions in Condition SEQ converged towards the welfare-maximizing equilibrium. In the augmented network rounds, subjects in every session did not converge to any stable equilibrium pattern – choices and traffic patterns remained fluctuating till the very end (see also the analysis of dynamics in Section 5.4).

In the basic network rounds, under Condition SIM, successful coordination to achieve the welfare-maximizing or most efficient equilibrium will result in each player incurring a travel cost of 15, while in the augmented network the welfare-maximizing equilibrium travel costs are 19 and17 (Table 1) – an increase in travel cost of 20%. The corresponding increase under Condition SEQ would be 15 to 19.5, a 30% increase. Yet the observed increases in individual travel cost upon network augmentation were 32% and 46% under Conditions SIM and SEQ, respectively, as stated in the previous subsection. This is all the more intriguing given that the two networks have the same number of (pure- and mixed-strategy) equilibria in Condition SIM as well as Condition SEQ, according to Table 1; yet coordination was far more successful in the basic network rounds than in the augmented network rounds in both conditions.

Here we point out a number of observations that may help us to understand the coordination failure in the augmented network rounds. Our observations are related to the fact that, as shown in Figure 3, the choice distributions in the augmented network rounds fluctuated through each session. In general:

(a) There were more choices of OABD than of OD on average (also reflected in Table 2). The average ratio of choice of OABD: choice of OD was approximately 11:7 or 10:8. This is not consistent with any of the equilibria in Table 1; as reported earlier, in Condition SIM these route choice distributions deviated significantly from all equilibrium predictions. Overall, the observations can be intuited via the cost difference function:

$$\Delta(r) = \left[\frac{90}{17 - r + 1} + 10\right] - [2(r+1)+1],$$

which indicates how much travel cost a player has to incur additionally by choosing OD over OABD, given that r other players choose OABD and the rest choose OD. Figure 4 is a smoothed plot of this function over the entire relevant range of r (= 0 to 17). As it appears, choosing OABD is always optimal except for a small range of r (= 9, 10, 11, 12, 13); even more importantly, there is relatively little to lose by choosing OABD over OD even when OD is the optimal choice (the maximum loss is 2.14, at r = 11), while choosing OD could lead to a maximum loss of 63 (at r = 17) when OABD is optimal. This phenomenon is driven by the fact that as r increases (so that the number of players on OABD decreases) *both* OD and OABD become more attractive, yet the marginal improvement for OD diminishes with r (an intrinsic characteristic of cost sharing) while that for OABD remains steady (the congestion cost function being linear). Thus, to a subject in our experiment, choosing OD would often be perceived considerably riskier than choosing OABD.

[Insert Figure 4 about here]

(b) In two sessions in Condition SIM (the top and bottom panels in the right column of Figure 3(a)), there was a transient phase in which a predominant number of subjects chose OABD, which reflected the feasibility of an equilibrium with all players traversing OABD under that condition (Table 1(a)). Nevertheless, further on in the session the OABD/OD choice disparity diminished, as subjects apparently attempted to migrate away from that very inefficient equilibrium.

Taken jointly, points (a) and (b) suggest that the tension between the high risks in choosing OD over OABD (point (a)) and subjects trying to coordinate to achieve as efficient an outcome as possible (point (b)) must have driven behavior towards the fluctuating patterns in Figure 3. In fact, the experimental outcome could be interpreted as a deviation from the equal split between OD and OABD that is an equilibrium under both conditions (Table 1(a) and (c)), the deviation having been caused by the perceived relative riskiness of OD over OABD.

5.4. Dynamics

5.4.1. Learning Effects. While the dynamics of play are not the primary focus of our study, we have carried out a series of related analysis (details can be found in Appendix C). Our approach involves dividing each experimental session into blocks of 10 rounds each, and then aggregate route choice distributions and travel costs within each block in each session. Further aggregation across sessions yields the block averages in Table 2. Our major conclusions are that:

(a) In Condition SIM, significant learning took place from block 1 to block 2 in the basic network rounds. There is also inconclusive evidence of learning from block 1 to block 3 in the augmented network rounds;

(b) In Condition SEQ, significant learning took place only from block 1 to block 2 in the basic network rounds.

Otherwise, there seemed to have been little learning in the experiment. That is, in the basic network rounds subjects' strategies did not change significantly overall after they had played about 10 rounds or so; learning was even less distinctive in the augmented network rounds. Notably, this conclusion is consistent with an inspection of the route choice evolutions in Figure 3, as well as the block averages in Table 2. The lack of learning in the augmented network rounds is in stark contrast with Rapoport et al. (2009)'s Experiment 1, in which there was substantial learning over the 40 rounds of the augmented network.

5.4.2. Strategic Teaching in the Basic Network Rounds. In the basic network rounds, all but one of the sessions in Condition SIM and all sessions in Condition SEQ converged towards the welfare-maximizing equilibrium. In Condition SEQ, this can be understood by the fact that the welfare-maximizing equilibrium is the unique equilibrium in the basic network (Table 1(c)); in fact, the converging trends from those sessions were apparent from the first round onwards (see Figure 3(b) and also Table A2 in Appendix C).

In Condition SIM, the situation was more complicated, especially because there are multiple equilibria for the basic network. The route choice frequencies of the sessions did not start out overwhelmingly in favor of OD (Figure 3(a) and Table A2 in Appendix C). Chi-squared analysis shows that these initial choice frequencies did not differ significantly between sessions (with $\chi^2(8) = 11.3$, p = 0.18); yet, one of the sessions converged towards very different traffic compared with the others, and as a result did not exhibit BP-type welfare loss when the session transitioned to the augmented network. As stated earlier, such behavioral phenomena can be related to theoretical studies of bifurcation in network games (Dal Forno and Merlone 2013a). They may also be accounted for in terms of different extent of strategic teaching (see Liu et al. 2015 and the references therein). With strategic teaching, a farsighted subject chose a route (OD) with poor short-term payoff in order to shift group decisions to the most efficient equilibrium and thereby increase her own long-term benefit.

The following analysis demonstrates the difference in the extent of strategic teaching between sessions that converged towards different equilibria in the basic network rounds. Across the four sessions in Condition SIM that converged towards the welfare-maximizing equilibrium in the basic network, there were 31 observations (out of 2808) of a subject facing a situation in which the best response to other subjects' choices in the previous round was *not* OD; moreover, they all appeared within the first 10 rounds. Among these, 19 of the observed choices were OD, indicating strategic teaching. That is, in those sessions, there were relatively very few opportunities for strategic teaching, and they all appeared early in the session; however, whenever such an opportunity arose, the subject often exhibited strategic teaching.

The situation was drastically different in the single session that did not converge towards the welfare-maximizing equilibrium: there were 362 opportunities (out of 702 observations) for strategic teaching in the basic network rounds, while among them, there were only 13 observed choices of OD; if we limit analysis to within the first 10 rounds (effectively round 2 to round 10), the numbers are 90 opportunities (out of 162 observations) among which only 11 observed choices of OD. Thus, there was relatively little strategic teaching in this session from early on.

5.5. Sequential Choice Dependence in Condition SEQ

We complete Section 5 with an analysis of sequential choice dependence in Condition SEQ. Recall that the subjects in that condition were randomly lined up in a queue to choose a route with full observability of predecessors' choices. Hence, the question naturally arises as to how (if at all) their choices depended on predecessors' choices.

To proceed, we examine whether a subject's choice of OD could be affected by: (a) the number of subjects ahead of her in the queue who chose OD, notated as n_{OD}; (b) the subject's position in the queue, notated as k; (c) in the augmented network, the number of subjects ahead of her in the queue who chose OABD, notated as n_{OABD}. We then carry out logistic regression of a binary dependent variable that is equal to 1, if the subject chose OD, and 0, otherwise, on the aforementioned independent variables as well as related interaction terms. Since decisions of the same subject, as well as subjects within the same session, were potentially correlated, we obtain our estimates using the Generalized Estimating Equations (GEE) approach (Hardin and Hilbe 2003). Our results are summarized in Table 3, where we have also included the Quasi-likelihood Information Criterion (QIC) goodness-of-fit statistics that is appropriate for GEE.

[Insert Table 3 about here]

Table 3 shows a significantly positive intercept for the basic network, reflecting the feasibility of the welfare-maximizing traffic as an equilibrium in that network; correspondingly, the intercept becomes non-significantly different from zero in the augmented network. Next, consistently across conditions, Table 3 shows that the choice of OD was more likely if more subjects in the queue had already chosen OD (i.e., a positive coefficient for n_{OD}). In the augmented network, the choice of OD was also more likely if more subjects in the queue had already chosen OABD (i.e., a positive coefficient for n_{OABD}). These results are intuitive as they reflect the positive network externalities of cost sharing on OD and the negative network externalities of congestion on OABD.

Note that n_{OD} in the basic network and $n_{OD} + n_{OABD}$ in the augmented network are effective proxies for the position of the subject in the queue, since there were few choices of OAD and OBD. However, the regression analysis also picks up effects of k, the subject's own position in the queue, over and above effects of n_{OD} and n_{OABD}. That is, a subject was less likely to choose OD the further back in the queue he/she was (a negative coefficient for k); in addition, the effect of n_{OD} in the augmented network diminished as the subject's position in the queue became further back (i.e., a negative coefficient for the interaction $k \times n_{\mathrm{OD}}$).

These effects may be interpreted as reflecting a concern of our subjects that, all else being equal, a subject's choice of OD could cause less bandwagon effect the further back the subject was in the queue. For example, if the subject was in the 11$^{\text{th}}$ position, compared with if she was at the 3$^{\text{rd}}$ position, choosing OD might have less effect on attracting subjects behind her to choose OD, because there would be fewer subjects behind in the former case. This possibility is contrary to the equilibrium predictions (Table 1(c)), which prescribe that the first nine subjects choose OABD and the last nine subjects choose OD, so that subjects further back in the queue would be more likely to choose OD. As a result, in the augmented network rounds across all sessions in Condition SEQ, 42.0% of route choices in the front half of the queue (position 1 to 9) were OD, while 40.7% of route choices in the back half of the queue (position 10 to 18) were OD. These figures deviate drastically from the equilibrium prediction of 0% in the front half and 100% in the back half.

6. Conclusions

Our study demonstrates, both theoretically and behaviorally, important extensions of the Braess Paradox in domains not covered in related literature. The existence of the cost-sharing route, with its positive network externalities, has the potential to attract players to achieve a highly efficient outcome. Yet the welfare-maximizing traffic is not even an equilibrium once the congestible part of the network is augmented by a single costly link.

It needs to be re-emphasized that the BP reported in this study has a fundamentally different characteristic from previously studied settings of BP. In previous BP settings such as Rapoport et al. (2009), BP occurred because the (typically unique) equilibrium changed. In the present study it happened partly because of coordination failure as a behavioral phenomenon.

In Condition SIM, the equilibrium changes caused by the augmentation of the network would not have resulted in as drastic a deterioration in welfare, if users (see Table 1(a)) coordinated well in both networks to achieve the most efficient (Pareto dominant) equilibrium in each network. Had that happened, the individual travel cost in Condition SIM would have increased mildly from 15 to 18, a 20% increase. Instead, in our experiment, average individual travel cost increased by 32% when the network was augmented in Condition SIM (Section 5.2).

In Condition SEQ, the high degree of choice observability might be expected to enhance players' coordination in achieving high efficiency as it eliminates the equilibrium multiplicity under the simultaneous protocol. Specifically, the sequential protocol in our study guarantees the welfare-maximizing traffic as the unique equilibrium in the basic network. The protocol eliminates the most inefficient equilibrium of all traversing OABD in the augmented network, though it cannot establish the welfare-maximizing traffic as an equilibrium in that network. As can be seen in Table 1(c), if high observability it improved coordination in both, the difference in individual travel cost in equilibrium between the two networks would be 4.5 on average, a 30% increase. However, behaviorally, the sequential protocol led to almost perfect convergence to equilibrium *only* in the basic network rounds, but this was far from the case in the augmented network rounds (Figure 3(b)); this led to further welfare loss in the latter network. The observed increase in individual travel cost from the basic to the augmented networks was 46% (Section 5.2). Overall, our experiment shows that choice observability could hardly improve the loss of welfare in our setting of the BP: both Conditions SIM and SEQ yielded similar increase in travel costs as the network was augmented.

As discussed earlier, our experiment has been intentionally designed to have subjects choose routes in the augmented network *after* choosing routes multiple times in the basic network, with the possibility that they might have been able to establish habitual choice patterns to mitigate the BP. This turned out not to be the case: there were substantial switches to OABD once the network was augmented, leading to a speedy deterioration of welfare among the players – compare, for example, the mean travel costs in the last block in the basic network rounds and the first block in the augmented network rounds in Table 2.

6.1. Future Directions

The present study is designed to demonstrate the theoretical and behavioral *feasibility* of the BP in the presence of mixed externalities and various levels of choice observability. It can serve as a baseline paradigm to be developed in different directions.

Overall, further theoretical analysis and experimentation might be considered to explore the general conditions by which the phenomena we observed could occur. We have focused on a simple network with link costs that are solely influenced by the traffic flow via simple cost functions; hence the costs over all routes can easily be enumerated. This setup has been partially motivated by the need to create an experimentally manageable context that subjects could grasp easily. In practice, many other factors could influence the link costs, notably exogenous incidental factors such as weather. In addition, there is the need to study network structures with richer architecture (e.g., see Roughgarden 2005 for general theoretical treatments of the BP with only negative externalities).

Elements of heterogeneity and incomplete information among users might also be introduced to explore an even wider range of circumstances. Coordination could be as much driven by the network and cost structures, as by network users' beliefs and behavioral factors such as trust, reciprocity (positive and negative), forgiveness, etc., that emerge during their interactions; these effects could also exhibit a high degree of individual heterogeneity. Future research may seek to understand them through more interventionist measurements such as belief elicitations

Lastly, we note that the single session in Condition SIM that did not exhibit the BP indicates bifurcations in the lab (Dal Forno and Merlone 2013a). Our setup was not aimed at studying bifurcation behavior, and the basic network was designed to render the choice of OD favorable. For example, it took only three or more subjects choosing OD to make the net payoff positive for that choice, and for any subject, as long as at least six other subjects chose OD, that route would be preferable to the others (see Appendix A). Therefore, it would be worthwhile to explore cost structures that are conducive to prominent behavioral bifurcation in our network context.

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Table 1. Equilibria in the Networks and Protocols in the Present Study (see Figure 2), with Comparisons From Rapoport et al. (2009)'s Games 1A and 1B (see Figure 1). Each five-component vector entry specifies an equilibrium traffic with the numbers of players traversing each route listed in the order OD, OAD, OBD, and OABD, followed by the individual travel cost(s). In entries with two costs separated by a forward slash (/), the first refers to players choosing OD and the second refers to players choosing OABD. Note that the costs in Rapoport et al. are scaled by a factor of 10 relative to the present setup.

Network type	Equilibrium type	Present study			Rapoport et al. (2009), Games 1A and 1B		
		Simultaneous protocol	Sequential protocol	Welfare Maximizing	Simultaneous protocol	Sequential protocol**	Welfare Maximizing
Basic	Pure-strategy	(18,0,0,NA,15) (0,9,9,NA,30)	(18,0,0,NA,15)	(18,0,0,NA,15)	(NA,9,9,NA,300)	(NA,9,9, NA,300)	(NA,9,9, NA,300)
	Mixed-strategy	(4.9,6.5,6.5,NA,28.2) (0,9,9,NA,30.5)	–		(NA,9,9, NA,305)	–	
Augmented	Pure-strategy	(10,0,0,8,19/17) (9,0,0,9,20/19) (0,0,0,18,37)	(9,0,0,9,20/19)*	(16,0,0,2,14.4)	(NA,0,0,18,360)	(NA,0,0,18,360)	
	Mixed-strategy	(5.3,0,0,12.7,27.1)	–		–	–	

* In this equilibrium, the first nine players in the sequence choose OABD and the remaining nine players choose OD.

** Rapoport et al. (2009) did not apply the sequential protocol.

Table 2. Mean Choice Frequencies and Travel Cost by Blocks of 10 Rounds Each, and Selected Equilibrium Predictions for Comparison (see Table 1). Also listed are the relevant overall observed means and equilibrium predictions in Rapoport et al. (2009)'s Experiment 1. Note that the costs in Rapoport et al. are scaled by a factor of 10 relative to our setup. The row "WM" lists the welfare maximizing traffic flows.

	Basic network				Augmented network				
Round	OD	OAD	OBD	Cost	OD	OAD	OBD	OABD	Cost
Condition SIM (simultaneous protocol; 40 rounds per network condition)									
1-10	10.9	3.50	3.64	21.7	6.70	0.52	0.48	10.3	24.2
11-20	13.9	2.08	1.98	18.5	4.98	0.32	0.34	12.4	27.7
21-30	14.1	1.88	1.98	18.3	7.24	0.04	0.04	10.7	23.1
31-40	14.3	1.80	1.88	18.4	7.88	0.00	0.04	10.1	21.5
Overall	13.3	2.32	2.37	19.2	6.70	0.22	0.23	10.9	24.1
Equil. (I)	18	0	0	15	10	0	0	8	18
Equil. (II)	0	9	9	30	9	0	0	9	19.5
WM	18	0	0	15	16	0	0	2	14.4
Rapoport et al. (2009)'s Experiment 1 (simultaneous protocol; 40 rounds per network condition)									
Overall	NA	9.02	8.98	305	NA	1.72	1.47	14.8	339
Equil.	NA	9	9	300/305	NA	0	0	18	360
WM	NA	9	9	300	NA	9	9	0	300
Condition SEQ (sequential protocol; 20 rounds per network condition)									
1-10	16.8	0.48	0.70	15.9	7.10	0.12	0.22	10.6	23.4
11-20	17.8	0.10	0.14	15.2	7.78	0.02	0.02	10.2	21.8
Overall	17.3	0.29	0.42	15.5	7.44	0.07	0.12	10.4	22.6
Equil.	18	0	0	15	9	0	0	9	19.5
WM	18	0	0	15	16	0	0	2	14.4

Table 3. GEE Regression Results for Condition SEQ. The dependent variable is a binary choice variable that is equal to 1, if the observed choice was OD, and 0, otherwise. Each entry is the regression coefficient of the corresponding independent variable with standard error in parentheses; where an estimate is significantly different from zero, it is marked by one or two asterisks (* $p < 0.05$; ** $p < 0.01$).

	Basic network	Augmented network
# Obs	1,800	1,800
Intercept	3.23 (0.36)**	0.53 (0.35)
k	-0.42 (0.09)**	-0.88 (0.29)**
n_{OD}	0.67 (0.21)**	1.16 (0.28)**
$k \times n_{OD}$	-0.01 (0.01)	-0.04 (0.01)**
n_{OABD}	NA	0.82 (0.29)**
$k \times n_{OABD}$	NA	-0.00 (0.01)
$n_{OD} \times n_{OABD}$	NA	0.01 (0.03)
$k \times n_{OD} \times n_{OABD}$	NA	0.00 (0.00)
QIC	577	2413

Figure 1. The Networks in Rapoport et al. (2009)'s Games 1A and 1B (which were used in their Experiment 1), with Cost Function on Each Link Indicated (n = 18)

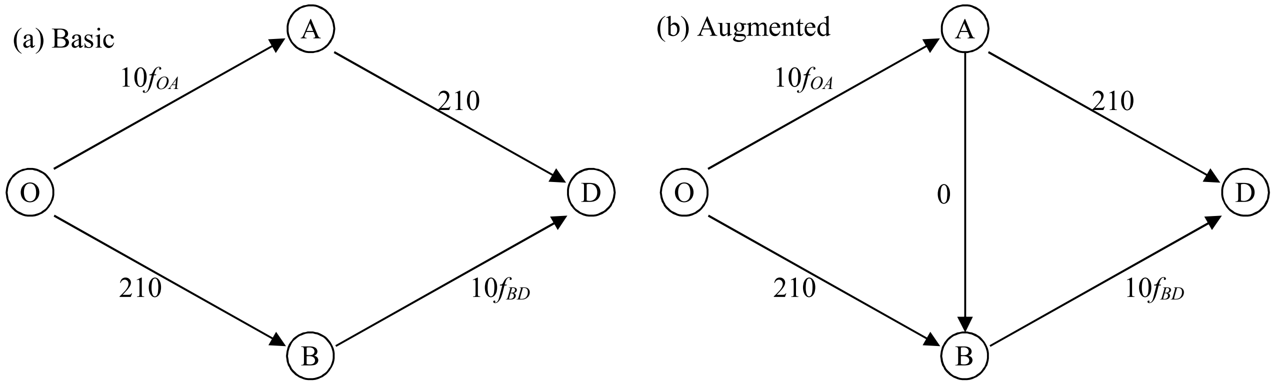


Figure 2. The Networks Used in the Present Study, with Cost Function on Each Link Indicated (n = 18)

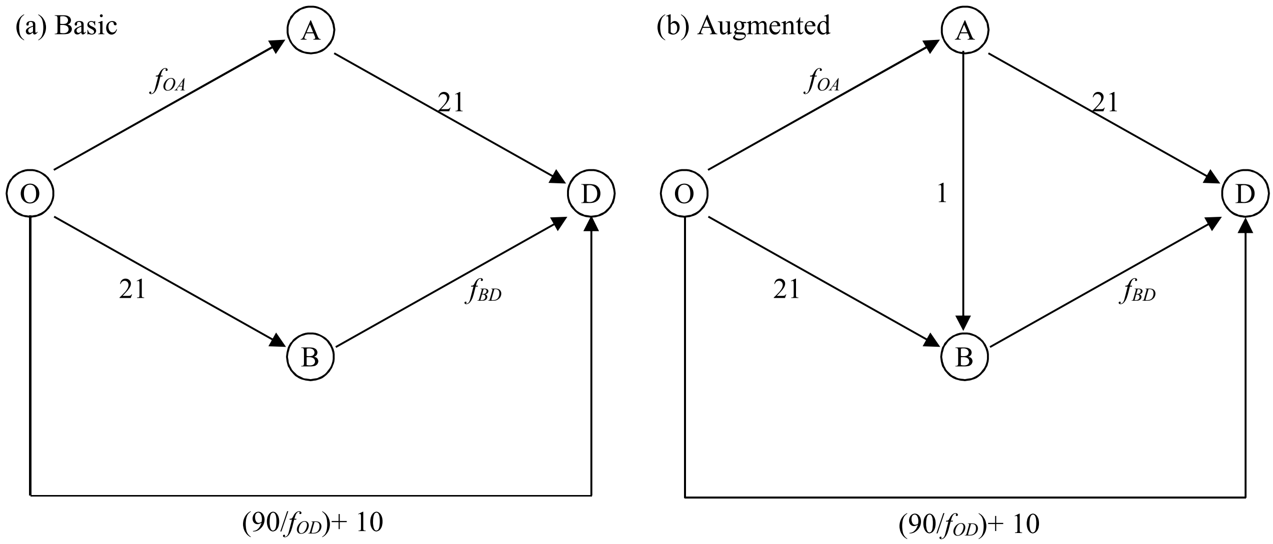


Figure 3. Number of Subjects Choosing Each Route, by Session and Network. Graphs in the Same Row are from the Same Session. Left Column: Basic Network; Right Column: Augmented Network

OD OAD OBD OABD

(a) Condition SIM (Simultaneous Protocol; 40 Rounds for Each Network)

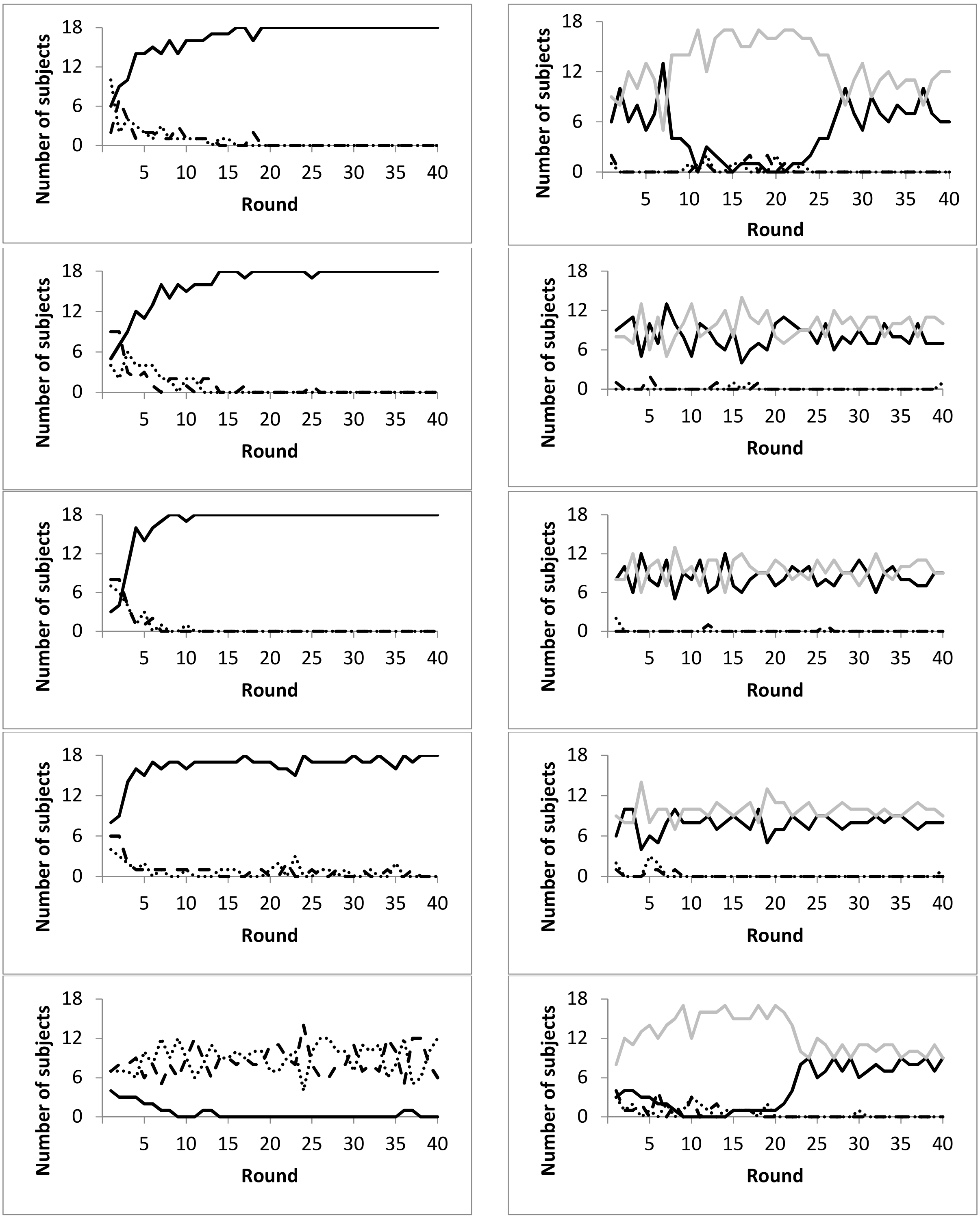


Figure 3 (cont'd)

(b) Condition SEQ (Sequential Protocol; 20 Rounds for Each Network)

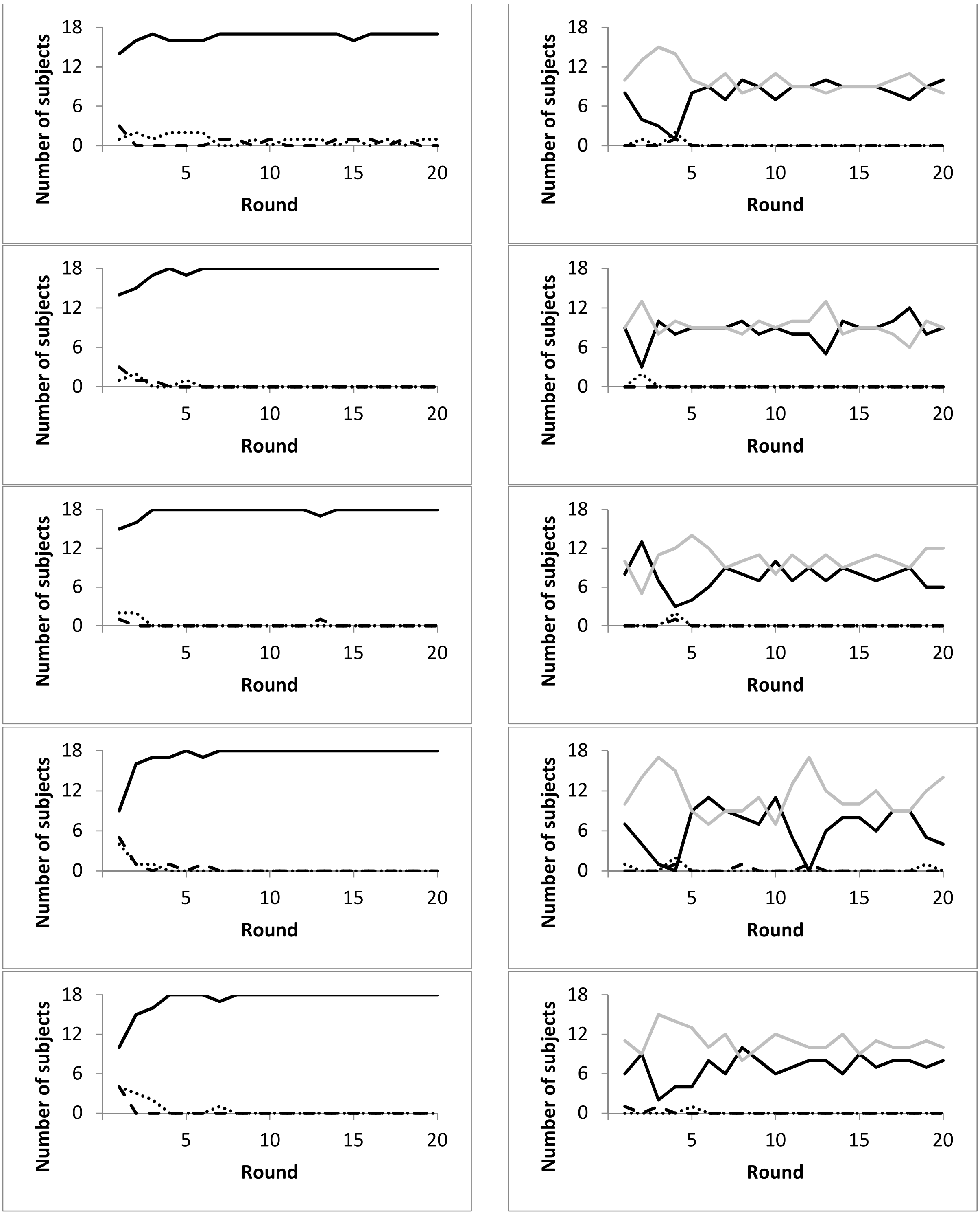
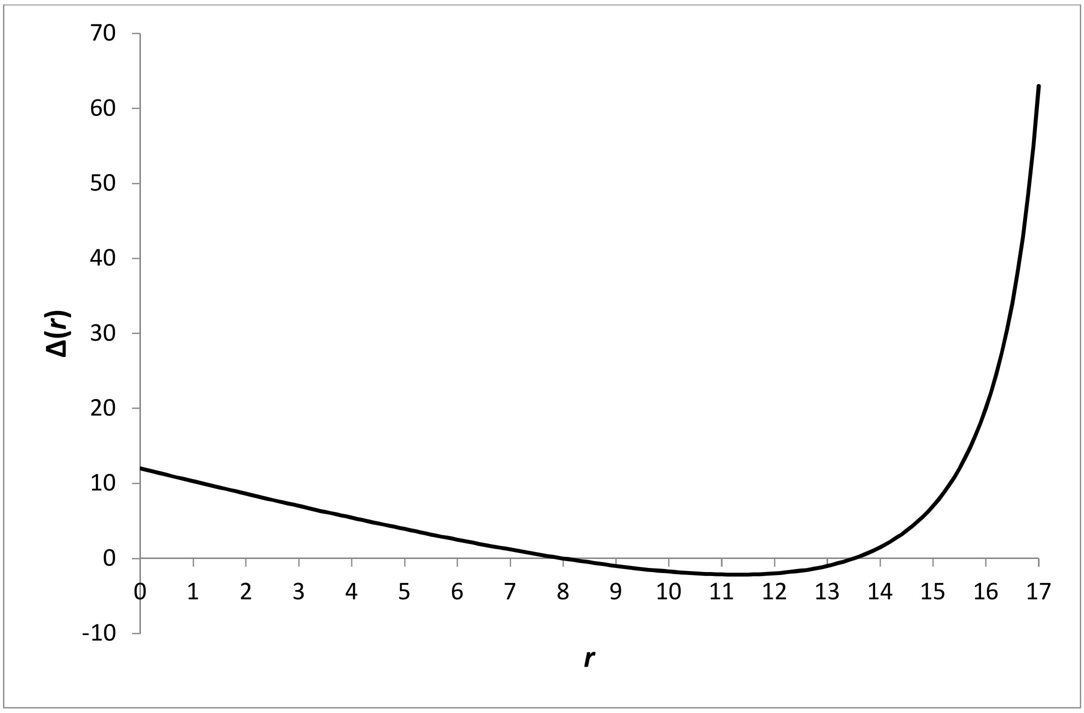


Figure 4. Smoothed Plot of the Cost Difference Function $\Delta(r)$ over $r = 0$ to 17



Appendix A

Derivation of Equilibria

1. Simultaneous Protocol (Condition SIM)

For the basic network, the numbers of players traversing OAD and OBD in any pure-strategy equilibrium cannot differ by more than one. Hence, any pure-strategy equilibrium traffic for the basic network under the simultaneous protocol must have the form:

$f_{OD} = s$;

$f_{OAD} = f_{OBD} = (18- s)/2$, if s is even;

$f_{OAD} = f_{OBD} + 1 = (19- s)/2$ or $f_{OAD} = f_{OBD} - 1 = (17- s)/2$, if s is odd,

where s may only be an integer in the set $\{0, 1, 2, 3, \ldots 18\}$. Numerically testing all these possibilities yields the first two equilibria listed in Table 1(a), which are simply either the traffic flow with the 18 players being splitted equally between OAD and OBD, or the welfare-maximizing traffic with all players traversing OD.

Symmetric mixed-strategy equilibria in the basic network under the simultaneous protocol must assign equal probabilities to choosing OAD and OBD; otherwise, for any individual player, the expected costs of choosing these two routes would not be equal, violating the definition of mixed-strategy equilibrium. Suppose that, in such an equilibrium, every player chooses OD with probability p and chooses OAD and OBD with equal probabilities $(1-p)/2$. If $1 > p > 0$, then we must have:

$$21 + 1 + 17 \cdot \frac{1-p}{2} = 10 + \sum_{s=0}^{17} \left[C_s^{17} p^s (1-p)^{17-s} \cdot \frac{90}{(s+1)} \right].$$

Numerically solving the above equation yields the equilibrium listed in Table 1(b), in which each player chooses OD with probability 0.27 and the other two routes with equal probabilities. Note that the case of $p = 1$ is the welfare-maximizing traffic and not a mixed-strategy equilibrium. The case $p = 0$ implies choosing OAD and OBD with equal probabilities of 0.5, which is obviously an equilibrium traffic and is also listed in Table 1(b).

In the augmented network, OAD and OBD are strictly dominated by OABD and therefore are not chosen in any equilibrium. Any pure-strategy equilibrium must have the form:

$f_{OD} = s = 18 - f_{OABD}$,

where s is an integer in the set $\{0, 1, 2, 3, \ldots 18\}$. Numerically testing all these possibilities yields the last three equilibrium traffic flows listed in Table 1(a), where the number of players traversing OD is respectively 10, 9, and 0 in each case.

For the welfare-maximizing traffic in the augmented network, we first note that any traffic with some players choosing OAD or OBD cannot be welfare-maximizing, based on a dominance argument. Thus, the welfare-maximizing traffic can be obtained by maximizing the total cost function:

$$r \cdot [(90/r) + 10] + (18 - r) * [2(18 - r) + 1]$$

over $r = 1, 2, 3 \ldots 18$ (the total cost when $r = 0$ can easily be shown to be suboptimal). The solution has 16 players traversing OD and two traversing OABD, which is not an equilibrium in the augmented network. The intuition is that OABD has very low cost when there is little traffic on it, due to the fact that its cost is almost entirely driven by the negative externalities of congestion, unlike OAD and OBD. Therefore, any traffic with very few or zero players on OABD (as is the case with the welfare-maximizing traffic) would attract switching from players on other routes, and cannot be an equilibrium.

Symmetric mixed-strategy equilibria in the augmented network must assign zero probability to OAD and OBD, since they are strictly dominated by OABD. Suppose that, in such an equilibrium, every player chooses OD with probability p and chooses OABD with probability (1-p). If $1 > p > 0$, then we must have:

$$1 + 2 \cdot [1 + 17(1 - p)] = 10 + \sum_{s=0}^{17} \left[C_s^{17} p^s (1 - p)^{17-s} \cdot \frac{90}{(s + 1)} \right].$$

Numerically solving the above equation yields the equilibrium traffic listed in Table 1(b), in which each player chooses OD with probability 0.29 and OABD with probability 0.71. Note that the cases of $p = 0$ or 1 are potential pure-strategy equilibria and have already been considered.

2. Sequential Protocol (Condition SEQ)

Consider first the basic network, and the choice of the last (18^{th}) player in the sequence. Observe that the last player should choose OD if there are already at least seven players on OD, regardless of the distribution of the remaining preceding players on the other routes; this is because, within our setting:

$$10 + \frac{90}{s+1} < 1 + 21$$

if and only if s, the number of players already on OD, is at least seven. This means that the 17^{th} player should choose OD if there are at least six players on OD already. Inducing further backward, we conclude that the 11^{th} player and all the players before him/her in the sequence should always choose OD regardless of their preceding players' choices. This implies that the unique subgame perfect equilibrium for the basic network is for all players to choose OD – the welfare-maximizing traffic – as listed in Table 1(c).

For the augmented network, we first reiterate the observation that OAD and OBD are strictly dominated by OABD. Hence, any subgame along the equilibrium path may only have positive number of players on OABD or OD. Consider the choice of the last player in the sequence. Given that there are already exactly s players on OD (so that there are already 17-s players on OABD), the last player should choose OD if:

$$10 + \frac{90}{s+1} < 1 + 2(17 - s + 1),$$

and OABD if the inequality is in the reverse direction; the player would be indifferent between the two if equality holds. Further calculations show that:

(i) The last player should strictly prefer OD if there are already four to eight players on OD, but not more or less (cf. Figure 4, where r in the figure denotes the number of other players on OABD, i.e., 17-s in the notation in this Appendix). The intuition is that, if there are too few players on OD, then cost-sharing is not sufficiently attractive; but if there are too many players on OD, then OABD becomes very uncongested and thus even more attractive than OD;

(ii) The last player would be indifferent between OABD and OD if there are already nine players on OD;

(iii) Otherwise, the last player should strictly prefer OABD.

This completely maps out all possible decision scenarios for the last player along the equilibrium path. We next induce backwards for the 17th player, and find that:

(i) The 17th player should choose OD if there are already three to seven players on OD;

(ii) If there are already eight players on OD (so that another eight players are already on OABD), the 17th player choosing OD would lead to the 18th player being indifferent between OD and OABD. If the 18th player chooses OD, the cost to the 17th player will be 19 (=10+(90/10)); if the 18th player chooses OABD, the cost to the 17th player will be 20 (= 10+(90/9)). If the 17th player chooses OABD, the 18th player will choose OD, and the cost to the 17th player will be 19 (= 1+ 2 × (8+1)). A trembling hand argument, assuming that the 18th player will choose OD and OABD with positive probabilities if he/she is indifferent between them, prescribes that the 17th player should then choose OABD;

(iii) The 17th player should otherwise choose OABD.

Inducing further backward, we conclude that the 10th player should choose OD if no player has chosen it yet, and all subsequent players will also choose OD; while the 9th player and all the players before him/her in the sequence should always choose OABD, regardless of their preceding players' choices. This implies that the unique subgame perfect equilibrium for the augmented network is for the first nine players to choose OABD and the last nine players to choose OD, as listed in Table 1(c).

3. Equilibria in the Networks in Figure 1 (Rapoport et al. 2009, Games 1A and 1B, Experiment 1)

As benchmarks for comparison, we discuss in this section the equilibria in the networks in Figure 1, which corresponds to Rapoport et al. (2009)'s Games 1A and 1B, that were used in their Experiment 1. The networks only have negative externalities and serve as the basis on which we construct our mixed externalities networks.

The equilibria under the simultaneous protocol are as described in Rapoport et al. (2009, Section 2). For the basic network, the pure-strategy equilibria have the players equally split between the two routes, with a travel cost of 300 each; the symmetric mixed-strategy equilibrium has every player choosing with equal probability between the two routes, with a travel cost of 305 each. For the augmented network, all players choose OABD with a travel cost of 360 each.

Rapoport et al. (2009) did not investigate the sequential protocol. For the basic network in their Experiment 1 under the sequential protocol, there may, in principle, be multiple subgame perfect equilibria involving different ways traffic build up on each route. However, all of them will end up with an equal split of players between the two routes with a travel cost of 300 each, like the pure-strategy equilibrium under the simultaneous protocol. Specifically, it is straightforward to see that if one route has nine players on it already, any players choosing after them will definitely choose the other route; therefore, no equilibrium will end up with more than nine players on any route, so that the only equilibrium must be the one with equal split. For the augmented network in their Experiment 1 under the sequential protocol, we reiterate the observation that OAD and OBD are strictly dominated by OABD. Thus, the only equilibrium is, as with the simultaneous protocol, for all the players to choose OABD with a travel cost of 360 each.

Appendix B

Sample Instructions (Condition SEQ)

Route Choice Experiment

Introduction

Welcome to our laboratory experiment on route choice in traffic networks. During this experiment you will be asked to make a large number of decisions and so will the other participants. Your decisions, as well as the decisions of the other participants, will determine your monetary payoff according to the rules of the network game that will be explained shortly.

Please read the instructions carefully. If you have any questions, please raise your hand and one of the supervisors will come to assist you.

Note that hereafter communication between the participants is prohibited. If they communicate with one another in any shape or form, then the experiment will be terminated.

The Route Selection Task

The experiment is computerized. You will make your decisions by clicking on the appropriate buttons that will appear on your screen (*please do not use the keyboard*).

There are **18** participants in the lab today who will play together as a single group for the duration of this experiment.

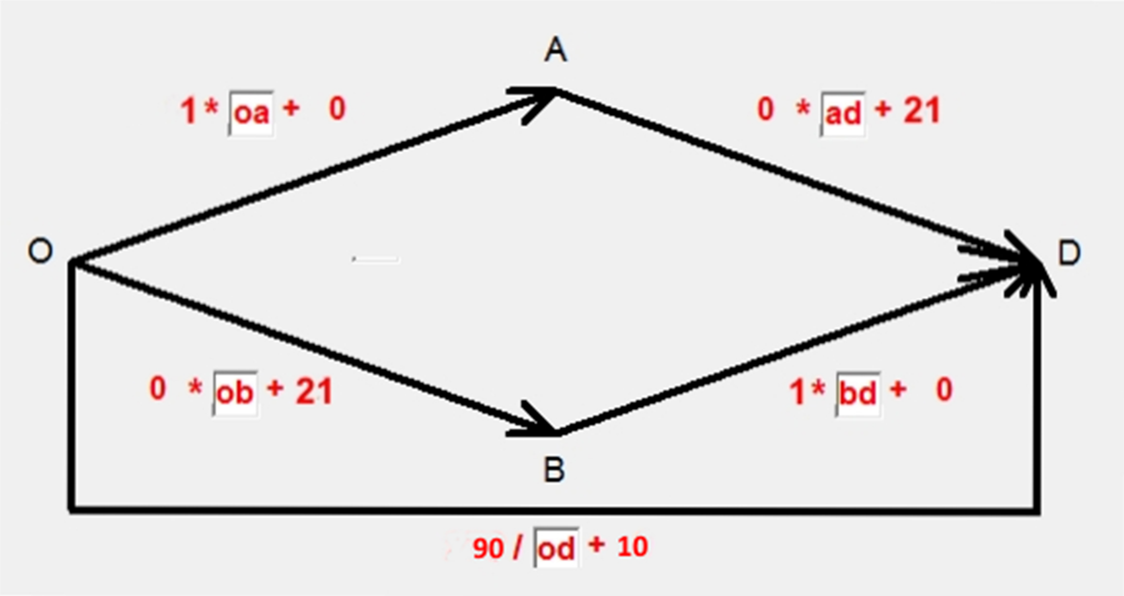
This experiment has two parts. Initially, you will participate in **20** identical rounds in Part I of a network routing game. On each round, you will be presented with a diagram of a traffic network with a single origin, a single destination, and **three** routes connecting them. Your task, as well as the task of the other participants, is to choose which route to traverse.

Once Part I is completed, all the group members will be handed the instructions for Part II.

Description of the Game

Consider the traffic network that is exhibited in the diagram below. You are required to choose one of **three** routes that connect the origin, denoted by the letter **O** (for Origin), to the final destination denoted by the letter **D** (for Destination). The 3 alternative routes in this traffic network are denoted in the diagram by the letter combinations [O-A-D], [O-B-D], and [O-D].

(*Please study this diagram.*)



Traversing a network is always costly in terms of the time needed to complete each segment of the road, gas, tolls, etc. The travel costs are shown near each segment of the routes that you may choose. For example, consider segment [O-A] (the top-left segment); each participant who chooses this segment will be charged a total cost of (1 × oa + 0) where 'oa' indicates the <u>number</u> of participants (that may range from 0 to 18) who choose this segment as part of their route. A similar cost structure applies to all the other segments in the network.

Please note that the cost charged for choosing each segment has two components. The first cost component is susceptible to congestion; it <u>changes</u> (increases or decreases) proportionally to the number of participants who choose that segment. The second cost component is not susceptible to congestion; it is <u>constant</u> and as such it is not affected by the number of participants choosing that segment.

For example, if only a single participant chooses segment [O-A], then his/her cost for this segment is $1 \times 1 + 0 = 1$. If, instead, 6 participants choose this segment, then the cost of this segment for <u>each</u> participant is $1 \times 6 + 0 = 6$. Similarly, if one participant chooses route [O-D], then his/her cost is $90/1 + 10 = 100$. If, instead, 9 participants choose route [O-D], then the cost for each of them is $90/9 + 10 = 20$.

At the end of *each round*, you will receive a <u>reward</u> of **45** tokens for reaching the destination. Your earnings will be determined by <u>subtracting</u> your cost for the round from this reward.

All the participants choose routes departing from the origin **O** and arriving at the destination **D**.

Example:

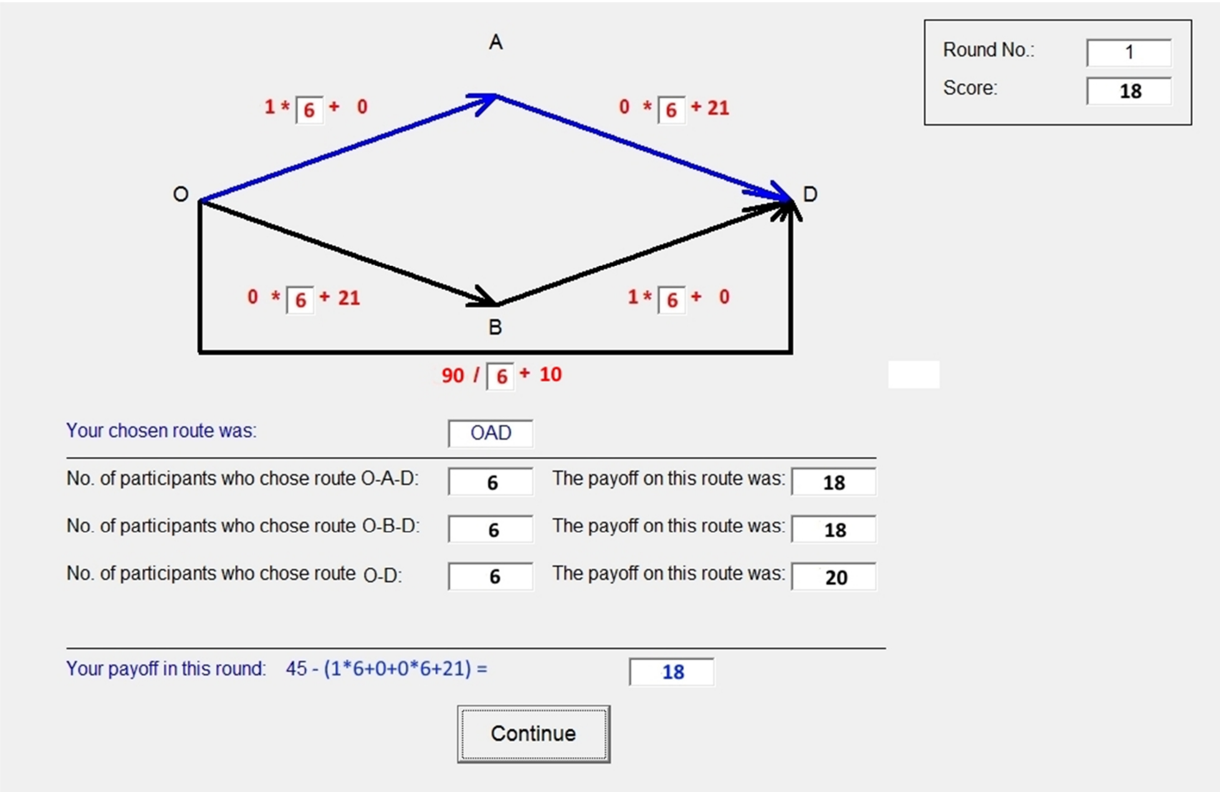
Suppose that on a certain round you choose route [O-A-D] and altogether, out of the 18 participants:

6 participants choose [O-A-D],

6 participants choose [O-B-D],

6 participants choose [O-D],

At the end of such a round, the computer will display the results as in the screen below:



The top part of the screen exhibits the three different routes and the number of participants who chose each route. The route that you have chosen is marked in blue.

The bottom part of the screen displays the number of participants who chose each route and the resulting payoff for the players doing it.

In this example, the total payoff that you earned by choosing route [O-A-D] was:

Cost of Segment [O-A]: $1 \times \mathbf{6} + 0 = 6$

Cost of Segment [A-D]: $0 \times \mathbf{6} + 21 = 21$

Earning: $45 - (6+21) = 18$ in this round.

The payoff for the other routes was calculated similarly.

If no participant chooses a given route, then the payoff will be denoted by N/A (for “not applicable”).

Note that the payoff on a certain route may be negative. For example, if **2** participants choose the route [O-D], then the payoff for choosing that route would be $45 - (90/\mathbf{2} + 10) = -10$.

You also will be provided with a History screen (*see example below*) that exhibits the number of participates choosing each route and the associated payoff in all previous rounds. You may open the History screen whenever you want.

	O-A-D		O-B-D		O-D	
Round	Players	Payoff	Players	Payoff	Players	Payoff
1	6	18	6	18	6	20

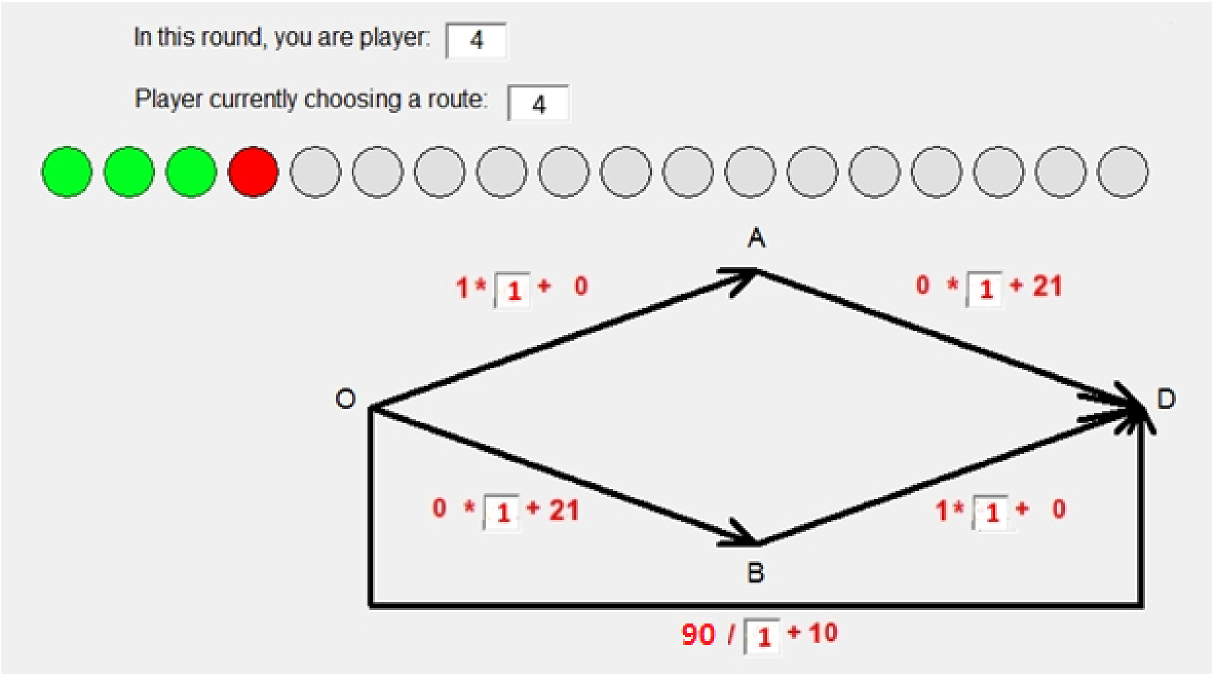
Back

All 20 rounds will have exactly the same structure.

<u>Procedure</u>

At the beginning of each round, the computer will display a network diagram similar to the one presented before. You will then be asked to choose one of the three possible routes. To choose a route, simply use the mouse to click on the segments comprising that route. The color of the segments that you choose will then change to blue indicating your choice. To change your choice, please click on the segment again and its color will change back to black. Once you are satisfied with your selection, please press the "Confirm" button. You will be asked to verify your choice.

Route choices will be made **sequentially one player at a time**. At the beginning of each round, you will randomly be assigned a number from 1 to 18 determining your position in the sequence. Please study the example below. The player in this example is designated to move 4th in the sequence as is indicated on top of the screen and also by the red dot.



The figure above shows that it is now the turn of the 4th player to choose a route (2nd row from the top). The dots of the first three players who already have chosen their routes appear in green. Once a player chooses a route, his dot will turn from red to green and the number of travelers on his route of choice will be incremented by 1. Studying the network in the example, notice that so far one player chose route [O-A-D], one chose [O-B-D], and a third player chose route [O-D]. Once player #4 chooses a route, her dot will turn green, and her choice of route will be recorded.

Once all the 18 participants confirm their choice of routes, the computer will exhibit a feedback screen similar to the one presented in the examples before.

It is vital that you keep track of the game progress and be ready to choose a route when it is your turn to play in order not to delay the progress of the experiment.

When you complete all 20 rounds in Part I, you will be handed another set of instructions for Part II.

Payments

At the end of the experiment, you will be paid for your cumulative earnings in all the 20 rounds in Part I as well as your cumulative earnings in all rounds of Part II. You will be paid in cash for your earnings with an exchange rate of $1 = **40** tokens. In addition, you will receive a show up bonus of $**5** for attending the experiment.

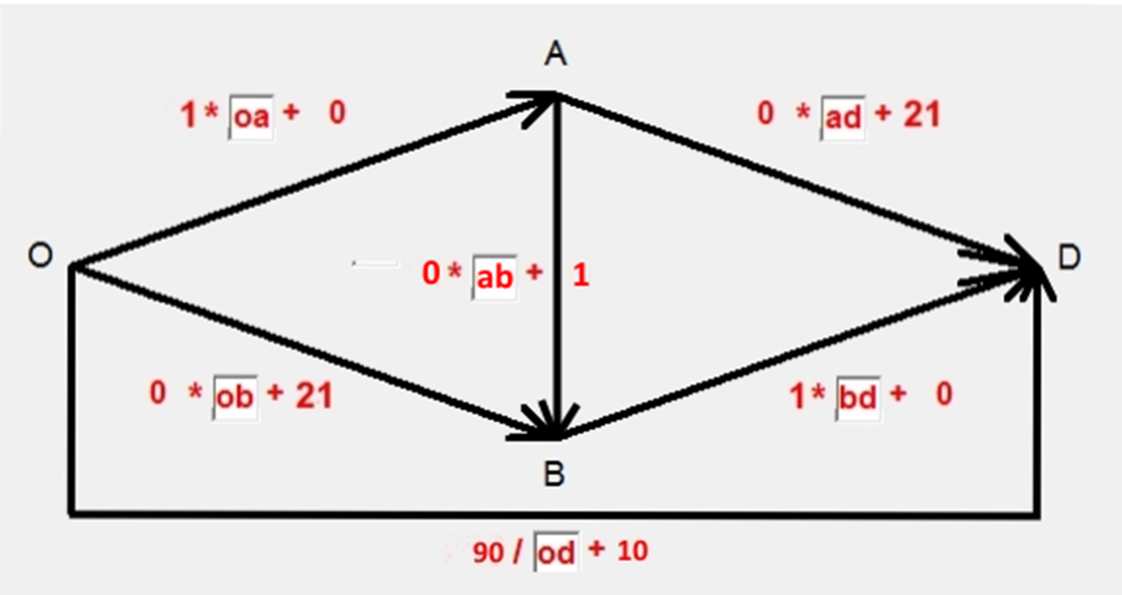
Once you are certain that you understand the task, please place the instructions on the table in front of you to indicate that you have completed reading them. If you have any questions, please raise your hand and one of the supervisors will come to assist you.

The experiment will begin shortly. Thank you for your participation.

Part II

Part II is identical to Part I with the exception that the traffic network has been expanded by adding a new road segment denoted by [A-B]. There is a constant cost of 1 token for traveling on this road segment (it has no congestible variable cost). Similar to Part I, you will be asked to choose a route from **O** to **D**. However, in Part II you have to choose one of **4** routes: the three original routes in the network presented in Part I, namely, routes [O-A-D], [O-B-D], and [O-D], as well as a new route [O-A-B-D] with three segments, namely, [O-A], [A-B], and [B-D]. Note that the new road segment [A-B] is directional and cannot be taken in reverse from B to A. Note, too, that route [O-A-B-D] shares the road segment [O-A] with route [O-A-D] and the road segment [O-B] with route [O-B-D].

The traffic network for Part II is displayed below. (*Please study this diagram.*)



Travel costs are computed exactly as in Part I. For example, a participant who chooses route [O-A-B-D] will be charged a total travel cost of (1 × oa + 0) + (1) + (1 × bd + 0), where 'oa' indicates that number of participants traversing segment [O-A] by choosing route [O-A-D] **or** route [O-A-B-D], and 'bd' indicates the number of participants traversing segment [B-D] by choosing route [O-B-D] **or** route [O-A-B-D].

Exactly as in Part I, at the end of each round you will receive a reward of **45** tokens for reaching the destination. Your earnings for that round will be determined by subtracting your travel cost from this reward.

As before, decisions will be made sequentially and at the end of each round a Feedback screen will be exhibited informing you of the routes chosen by all the participants and the resulting payoffs. You also will have the option of opening the History screen that shows the results of all the previous rounds in this part.

Payoffs will be determined exactly as in Part I, i.e., you will be paid for your cumulative earnings in all 20 rounds of Part II with the same exchange rates of $1 = **40** tokens. Therefore, at the end of the session you will be paid for your cumulative earnings across all the **40** rounds in Parts I and II.

Once you are sure that you fully understand the task please place the instructions on the desk in front of you to indicate that you have completed reading them. If you have any questions, please raise your hand and one of the experimenters will come to assist you.

Part II will begin shortly. Thank you for your participation.

Appendix C

Additional Data Analysis

1. Tests against Equilibrium Predictions

1.1. Route Choice Distributions

In the basic network rounds, nine out of the 10 sessions across both conditions converged towards the welfare-maximizing equilibrium. Since that equilibrium has all players choosing the same route, standard statistical tests of distributions, such as the chi-squared test, are not appropriate. Nevertheless, observably from Figure 3, the convergence was quite clear cut in every one of those nine sessions.

The one session in the basic network rounds that did not converge towards the welfare-maximizing equilibrium was in Condition SIM. That session converged towards traffic with virtually no subject choosing OD (see the bottom row of Figure 3(a)). The equilibrium traffic with such characteristics in the basic network rounds has all the route choices split equally across OAD and OBD on average (see Table 1(a) and 1(b)). Since there is only one independent session here, we cannot carry out the *t*-test analysis described in the next paragraph. Instead, for the final 10 rounds in the basic network rounds in that session, we test the conditional distribution hypothesis that, on average, among players who chose OAD and OBD, 50% chose the former and 50% chose the latter. Our hypothesis could not be rejected, $\chi^2(1) = 0.009$, $p >$ 0.9. We conclude that that session converged towards traffic that was not significantly different from the equilibrium traffic with equal split between OAD and OBD.

From Figure 3, it is clear that traffic in the augmented network rounds in every session converged towards a state in which no player chose OAD and OBD but were split between choices of OD and OABD. To proceed, for each session, we calculate the average number of subjects choosing OD over the final 10 rounds in the augmented network rounds. We then use these session averages as independent data points by which we employ *t*-tests to test against equilibrium predictions in Table 1 that have positive expected number of subjects on both OD and OABD. We find that, in Condition SIM, choice frequencies of OD in the final 10 rounds in the augmented network rounds were significantly different from all equilibrium predictions at $p < 0.01$. Similar *t*-tests with choice frequencies of OABD yield the same conclusion.

We next carry out the same analysis in Condition SEQ, where the only equilibrium prediction has traffic being split equally between OD and OABD in the augmented network rounds. Corresponding *t*-tests could not reject the equilibrium predictions for both choice frequencies of OD ($t(4) = -2.3$, $p = 0.08$) and OABD (($t(4) = 2.4$, $p = 0.07$).

Thus, we conclude that traffic in the basic network rounds converged towards the welfare-maximizing equilibrium of all choosing OD, apart from the exceptional session in Condition SIM in which it converged towards an equilibrium traffic with equal split of all subjects across OAD and OBD. By contrast, traffic in the augmented network rounds deviated significantly from all equilibrium predictions in Condition SIM, but converged towards the equilibrium traffic of equal split across OD and OABD in Condition SEQ. This is largely consistent with our general intuition that subjects in Condition SEQ would be relatively able to coordinate among themselves to achieve an equilibrium outcome.

1.2. Travel Costs

We also carry out tests on whether travel costs converged towards equilibrium predictions. For all but one sessions, the equilibrium prediction to test for the basic network rounds is 15 (the welfare-maximizing equilibrium cost). For each of those sessions, we calculate the average individual travel cost over the final 10 rounds in the basic network rounds. We then use these session averages as independent data points by which we employ *t*-tests to test against the predicted value of 15, separately for each condition (four sessions in Condition SIM and five sessions in Condition SEQ). We cannot reject the hypothesis that the travel costs were significantly different from 15 in both conditions (Condition SIM: $t(3)=1.00$, $p> 0.3$; Condition SEQ: $t(4)=1.11$, $p> 0.3$).

In the augmented network rounds, the average individual travel cost in equilibrium can be 18, 19.5, 37, or 27.1 in Condition SIM, according to Table 1(a) and 1(b). Using the same *t*-test approach for session average over the final 10 rounds under this network, we find that travel costs in Condition SIM were significantly higher than 18 and 19.5 but significantly lower than 27.1 and 37 in the augmented network rounds ($p < 0.01$ in all tests). The corresponding test for Condition SEQ is against the equilibrium prediction of 19.5 (Table 1(c)), and yields a non-significant difference ($t(4) = 2.37$, $p=0.08$).

To conclude, in the sessions that seemingly converged towards the welfare-maximizing equilibrium in the basic network rounds, the corresponding travel costs also converged towards the welfare-maximizing level. However, travel costs in the augmented network rounds did not converge towards any equilibrium level in Condition SIM, but did converge towards the equilibrium level in Condition SEQ.

2. Learning Effects

3.1. Session-level Learning

We divide each experimental session into blocks of 10 rounds each, and then aggregate route choice distributions and travel costs within each block in each session. Further aggregation across sessions then yields the block averages in Table 2. We then carry out, for the basic network under each condition, pairwise MANOVAs between every pair of successive blocks, with block means of choice frequencies of OD and OAD as the dependent variables. The tests reveal significant learning effects only between blocks 1 and 2 (Condition SIM: Wilks' $\lambda = 0.31$, $F(1,4) = 8.96$, $p < 0.05$; Condition SEQ: Wilks' $\lambda = 0.15$, $F(1,4) = 23.06$, $p < 0.01$). Block-by-block pairwise t-tests of block mean travel costs yield the same conclusions for learning over blocks 1 and 2 at $p < 0.05$ for both conditions. There is no other significant block-by-block learning effect with $p < 0.05$, nor in similar analysis for the augmented network rounds.

2.2. Individual Switching Frequencies

The above analysis pertains to learning at the level of the session. It is also instructive to analyze learning at the individual level. One approach is to look at the percentage frequencies by which individual subjects switched routes within each block of 10 rounds; that is, for each subject, we calculate the percentage of rounds within each block in which the subject switched routes from the previous round (if there was a previous round at all). Note that, in this analysis, the first block of the basic network rounds only included nine rounds of observations; meanwhile, the analysis of the first block of the augmented network rounds includes switching from the final basic network round.

It might be expected that subjects would switch routes less frequently as they played each game repeatedly. Indeed, this is borne out in the mean individual percentage frequencies in Table A1: switching became less frequent from block 1 to block 2 in all cases shown in the table; there was also a further decrease of switching from block 2 to block 3 in the augmented network rounds in Condition SIM. We proceed to verify these observations by within-subjects ANOVA pairwise comparisons for all pairs of consecutive blocks, with the mean individual percentage switching frequency of each session in each block as the dependent variable. The analysis reveals significant learning effects from block 1 to block 2 in Condition SIM in both networks, as well as in Condition SEQ in the basic network rounds, at $p < 0.05$. However, the learning effect from block 1 to block 2 in Condition SEQ in the augmented network rounds was only marginally significant ($F(1,4) = 6.61$, p=0.06). On the other hand, there is a significant learning effect from block 2 to block 3 in the augmented network rounds in Condition SIM ($F(1,4)$=76.1, $p < 0.01$). We observe no other significant learning effects in switching, as all other pairwise comparisons yield $p > 0.1$.

It is also worth noting a correlational analysis between the per subject average switching frequencies within a session and the corresponding average travel cost. In the basic network rounds, this correlation was significantly and highly positive in block 2 in Condition SEQ and in all blocks in Condition SIM; the correlation coefficients are all higher than 0.95 (with $p < 0.01$) in these cases. Otherwise (including all the augmented network blocks), the correlations are non-significantly different from zero (p>0.05). These results suggest that frequent switching was conducive to higher travel costs only when there was stable convergence to the welfare-maximizing equilibrium in the basic network; otherwise, frequent switching had no impact on costs.

The learning effects reported here suggest that, to be more rigorous in statistical testing on our data, we should exclude observations from the first 10 rounds of both networks in both conditions. Re-analysis with this exclusion does not alter our major qualitative conclusions. In particular, the mixed-design 2(choice observability: Condition SIM vs. Condition SEQ) × 2(network: basic vs. augmented) ANOVA employed earlier to test for statistical evidence of the BP still reveals a significant main effect in network ($F(1,8)$ =

16.6, $p < 0.01$) but no other significant effects at $p < 0.05$. That is, even if we consider only the rounds when choice distributions had stabilized, the BP remains evident.

Table A1. Mean Individual Switching Percentage Frequencies by Blocks of 10 Rounds Each. Note that a switch of route from the final basic network round to the first augmented network round counts as a switch within the augmented network round data

Round	Basic network	Augmented network
Condition SIM (simultaneous protocol; 40 rounds per network)		
1-10	30.5%	34.6%
11-20	12.4%	21.6%
21-30	10.1%	14.1%
31-40	9.7%	14.2%
Overall	15.7%	21.1%
Condition SEQ (sequential protocol; 20 rounds per network)		
1-10	6.4%	34.8%
11-20	1.2%	24.7%
Overall	3.8%	29.7%

Table A2. Choice Frequencies in the First Round of Every Session (When Subjects Played the Basic Network Game). The session numbers correspond to the panels in Figure 3(a) and (b), so that the top panel in Figure 3(a) is Session 1 of Condition SIM in the table, etc. The shaded row indicates the session that did not eventually converge towards the welfare-maximizing equilibrium

Condition	*Session*	*#OAD*	*#OBD*	*#OD*
SIM	1	2	10	6
	2	9	4	5
	3	8	7	3
	4	6	4	8
	5	7	7	4
SEQ	1	3	1	14
	2	3	1	14
	3	1	2	15
	4	5	4	9
	5	4	4	10